OR

B i) State and prove Binomial theorem.
ii) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(2x-3y)^{25}$?

23. A i) Solve the recurrence relation $a_k = 3a_{k-1}$, for k = 1,2,3 ... and the initial condition $a_0 = 2$.
ii) Use generating functions to show that $\sum_{k=0}^{n} C(n,k)^2 = C(2n,n)$ whenever n is a positive integer.

OR

B i) Find the generating functions for $(1+x)^{-n}$ and $(1-x)^{-n}$, where n is a positive integer using the extended Binomial theorem.
ii) Using generating functions to find an explicit formula for a_n where $a_n = 8a_{n-1} + 10^{n-1}$ which satisfies the recurrence relation and the initial condition $a_1 = 9$.

24. A Use k – maps to minimize the sum of the products expansion
i) $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z}$
ii) $x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$
iii) $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$

OR

B Use the Quine McCluskey method to simplify the sum – of – products expansion
$wxy\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$

25. A Find a regular expression that specifies each of these sets:
i) The set of bit strings with even length.
ii) The set of bit strings ending with a 0 and not containing 11
iii) The set of bit strings containing an odd number of 0's

OR

B Show that the set $\{0^n 1^n / n = 0,1,2 \ldots\}$ made up of all strings consisting of a block of 0's followed by a block of an equal number of 1's is not regular.

8|12|23

Four Pages
S. No. 70615

23PMAE08

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END SEMESTER EXAMINATION NOV/DEC-2023

First Semester

M.Sc MATHEMATICS

ELECTIVE – II DISCRETE MATHEMATICS

Time: Three Hours **Maximum: 75 marks**

SECTION A – (15 x 1 = 15 marks)
ANSWER ALL QUESTIONS

1. A ________ is a declarative sentence that is either true or false but not both.
A Statement B Logic
C Inference D Proposition
2. The steps of an algorithm must be defined precisely, it is called
A Finiteness B Effectiveness
C Definiteness D Generality
3. The big – O symbol is sometimes called a
A Paul Symbol B Landau Symbol
C Donald Symbol D Knuth Symbol
4. How many different bit strings of length seven are there?
A 128 B 64
C 32 D 0
5. If n be a non-negative integer, then $\sum_{k=0}^{n} 2^k \binom{n}{k} = ----$
A 3^n B 2^n
C 4^n D 1^n
6. The next larger 4 – combination of the set {1,2,3,4,5,6} after {1,2,5,6} is

A {1,2,3,4} B {1,3,4,5}
C {1,3,5,6} D {1,2,3,5}

7. The sequence $\{a_n\}$ with _____ is a solution of the recurrence relation.
A $a_n = \alpha_1 r_1^{-n} + \alpha_2 r_2^{-n}$ B $a_n = \alpha_1 r_1^{-n} + \alpha_2 r_2^{n}$
C $a_n = \alpha_1 r_1^{n} + \alpha_2 r_2^{n}$ D $a_n = \alpha_1 r_1^{n} + \alpha_2 r_2^{-n}$

8. If f(n) = 5f(n/2)+3 and f(1) = 7. $f(2^k)$ = ?
A $5^k\left(\frac{3}{4}\right) - \left(\frac{31}{4}\right)$ B $5^k\left(\frac{3}{4}\right) + \left(\frac{31}{4}\right)$
C $5^k\left(\frac{31}{4}\right) + \left(\frac{3}{4}\right)$ D $5^k\left(\frac{31}{4}\right) - \left(\frac{3}{4}\right)$

9. The function f(x) = 1/(1-x) is the generating function of the sequence ______.
A 0, 0, 0, 0, ... B 0, 1, 0, 1, ...
C 1, 0, 1, 0, D 1, 1, 1, 1, ...

10. The value of $1.0 + \overline{(0+1)}$ = ______.
A 1 B -1
C 0 D ∞

11. ______ is one of the Idempotent law.
A x + x = 0 B x + x = x
C x + x = 1 D x + x = -1

12. The product of literals corresponding to a black of all 1's in the k -map is called a/an
A Product B Implicant
C Minterms D Maxterms

13. A vocabulary V is a finite, non-empty set of elements called
A Symbols B Sentence
C String D None

14. The minimum state automation accepting a regular set L is unique up to an
A Homomorphism B Isomorphism

C Mesomorphism D Heteromorphic

15. The halting problem is an ______ decision problem.
A Solvable B Unsolvable
C Turing machine D Machine

SECTION B – (2 x 5 = 10 marks)

ANSWER ANY TWO QUESTIONS

16. Show that i) $7x^2$ is $O(x^3)$ ii) n^2 is not O(n)
17. How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2, and x_3 are non-negative integers?
18. Find the explicit formula for the Fibonacci numbers.
19. i) Prove that the absorption law x(x+y) = x using other identities of Boolean algebra.
ii) Find the sum of products expansion for the function $F(x,y,z) = (x+y)\bar{z}$
20. Construct a non-deterministic finite–state automation that recognizes the language generated by the regular Grammer G = {V, T, S, P), where V = {0, 1, A, S}, T = {0, 1} and the productions in P are S→1 A, S→0, S→λ, A → 0A, A →1 A and A→1.

SECTION C – (5 x 10 = 50 marks)

ANSWER ALL QUESTIONS

21. A i) Show that $p\vee(q\wedge r)$ *and* $(p\vee q)\wedge(p\vee r)$ are logically equivalent.
ii) Show that $(p\wedge q)\longrightarrow(p\vee q)$ is a tautology.

OR

B i) Use the insertion sort to put the elements of the list 3, 2, 4, 1, 5 in increasing order.
ii) Give a big- O estimate for f(x) = (x + 1) log $(x^2+1) + 3x^2$

22. A i) If n is a positive integer and r is an integer with 1 ≤ r ≤n, then prove that there are P(n, r) = n(n-1)(n-2)....(n-r+1).
ii) if n and r are integers with 0 ≤ r ≤ n, then prove that $P(n,r) = \frac{n!}{(n-r)!}$