B If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0 then prove that $W(\phi_1\phi_2)(x) = e^{-ax}(x - x_0)W(\phi_1, \phi_2)(x_0)$.

22. A If ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \cdots + a_n y = 0$ on an interval I containing a point x_0 prove that for all $x \in I$ $||\phi(x_0)||e^{-k|x-x_0|} \le ||\phi(x)|| \le ||\phi(x_0)||e^{k|x-x_0|}$ where $k = 1 + |a_1| + \cdots . + |a_n|$.

OR

B Solve $y''' + y'' + y' + y = 1$ which satisfies $\psi(0) = 0$, $\psi'(0) = 1, \psi''(0) = 0$.

23. A If one solution of $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. find the basis for the solution for $x > 0$.

OR

B Show that $\int_{-1}^{1} P_n(x)P_m(x)\,dx = 0, (n \neq m)$.

24. A Find all solutions of the following equation for $x > 0$ $x^3y''' + 2x^2y'' - xy' + y = 0$.

OR

B Show that $x^{\frac{1}{2}}J_{\frac{1}{2}}x = \frac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)}\sin x$.

25. A Find the solution of $\phi(x)$ of the first order equation $y' = xy, y(0) = 1$ by finding successive approximation solution.

OR

B If M, N be two real valued function which have continuous first partial derivatives on some rectangle $R: |x - x_0| \le a, |y - y_0| \le b$ prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

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END SEMESTER EXAMINATION NOV/DEC-2023

First Semester

M.Sc Mathematics

CORE – III ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours **Maximum: 75 marks**

SECTION A – (15 x 1 = 15 marks)
ANSWER ALL QUESTIONS

1. If $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + P(x)y' + Q(x)y = 0$, then.
   A $y_1(x) + y_2(x)$ B $c_1y_1(x) + y_2(x)$
   C $c_1y_1(x) + c_2y_2(x)$ D $y_1(x) + c_2y_2(x)$
2. General solutions of first order homogeneous equation $y' + P(x)y = 0$.
   A $y = e^{\int pdx}$ B $y = e^{-\int pdx}$
   C $y = \int pdx$ D $y = \log(\int pdx)$
3. The solution of the initial value problem $y'' + y = 0$, y (0) = 0, y' (0) = 1.
   A sin x + cos x B sin x + $c_2 \cos x$
   C $c_1 \sin x + \cos x$ D $c_1 \sin x + c_2 \cos x$
4. The form of the exact solution to $2\frac{dy}{dx} + 3y = e^{-x}$, y (0) = 5 is.
   A $Ae^{-1.5x} + Be^{-x}$ B $Ae^{-1.5x} + Bxe^{-x}$
   C $Ae^{1.5x} + Be^{-x}$ D $Ae^{1.5x} + Bxe^{-x}$
5. The n^{th} order ordinary linear homogeneous differential equations have.
   A n – singular solutions B no singular solution

C one singular solution D none of these

6. If the Legendre's equation $(1-x^2)\,y''-2xy'+p(p+1)y=0$, find $P(x)=$?

A $\frac{2x}{1-x^2}$ B $\frac{-2x}{1-x^2}$

C $\frac{2x}{1+x^2}$ D $\frac{2x^2}{1-x^2}$

7. If the product $(x-x_0)P(x)$ and $(x-x_0)^2\,Q(x)$ is analytic at $x=x_0$ then x_0 is said to be an ________ of $y''+P(x)y'+Q(x)y=0$.

A ordinary point B singular point

C regular singular point D irregular singular point

8. If the Bessel's equation is $x^2\,y''+x\,y'+(x^2-p^2)\,y=0$, then find $P(x)=$?

A x B $\frac{x^2-p^2}{x^2}$

C $\frac{1}{x}$ D $\frac{x^2}{x^2-p^2}$

9. The regular singular points for the differential equation $(1-x^2)\,y''+y'+y=0$.

A $x=1\ \&\ x=-1$ B $x=1\ \&\ x=1$

C $x=0\ \&\ x=-1$ D $x=1\ \&\ x=0$

10. The solution of $y'=y^2$ with initial condition $\phi(1)=-1$.

A x B x^2

C $1/x$ D $-1/x$

11. The homogeneous differential equation $M(x,y)\,dx+N(x,y)\,dy=0$ can be reduced to a differential equation, in which the variable is separated, by the substitution.

A $y=v\,x$ B $x\,y=v$

C $x+y=v$ D $x-y=v$

12. The integrating factor of the differential equation $\frac{dy}{dx}(x\log x)+y=2\log x$ is.

A e^x B $\log x$

C $\log(\log x)$ D x

13. The general solution of $\frac{dx}{dt}=2x$, $\frac{dy}{dt}=3y$.

A $x=c_1e^{2t}\ \&\ y=c_2e^{3t}$ B $x=c_1e^{-2t}\ \&\ y=c_2e^{-3t}$

C $x=c_1e^{t}\ \&\ y=c_1e^{-t}$ D $x=c_1e^{-3t}\ \&\ y=c_2e^{-2t}$

14. $y'=3y^{\frac{2}{3}}, y(0)=0$ does not satisfy Lipschitz condition on__________.

A $R:|x|\geq 1, |y|\leq 1$ B $|x|\geq 1, c\geq y\geq d$ where $0>c>d$

C $R:|x|\leq 1, |y|\leq 1$ D None of these

15. A solution of the differential equation $\frac{dy}{dx}=x+y, y(0)=1$ starting with $y_0(x)=1$ use Picard's method, First approximation value =

A $1-x$ B $1+x+\frac{x^2}{2}$

C $1-x+x^2-\frac{x^3}{3}+\frac{x^4}{4!}$ D $1-x+x^2$

SECTION B – (2 x 5 = 10 marks)

ANSWER ANY TWO QUESTIONS

16. Define Wronskian.
17. Solve $y'''-3y'+2y=0$.
18. Write the Legendre equation.
19. Define Euler equation.
20. Solve $y'=3y^{2/3}$.

SECTION C – (5 x 10 = 50 marks)

ANSWER ALL QUESTIONS

21. A Solve $y''-y'-2y=e^{-x}$.

OR