22. A If two of the three integrals in $\int_a^c f\ d\alpha + \int_c^b f\ d\alpha = \int_a^b f\ d\alpha$ exists prove that the third integral also exists.

OR

B If $\alpha \nearrow$ on $[a,b]$ then prove that the following three statements are equivalent i) $f \in R(\alpha)$ on $[a,b]$

ii) f satisfies Riemann's condition with respect to α on $[a,b]$

iii) $\underline{I}(f,\alpha) = \bar{I}\ (f,\alpha)$.

23. A If α be of bounded variation on $[a,b]$ and assume that $f \in R(\alpha)$ on $[a,b]$ then prove that $f \in R(\alpha)$ on every interval $[c,d]$ of $[a,b]$.

OR

B State and prove Lebesgue's criterion for Riemann-integrability.

24. A If a series is convergent with sum S then prove that it is also $(C,1)$ summable with Cesaro sum S.

OR

B State and prove Abel's limit theorem.

25. A If $f_n \to f$ uniformly on S and each f_n is continuous at a point c of S then prove that the limit function f is also continuous at c.

OR

B If $\lim_{n\to\infty} f_n = f$ on $[a,b]$ and $g \in R$ on $[a,b]$ define $h(x) = \int_a^x f(t)g(t)dt$, $h_n(x) = \int_a^x f_n(t)g(t)dt \ for\ x \in [a,b]$ then prove that $h_n \to h$ uniformly on $[a,b]$.

Four Pages

S. No. 70612

23PMAC02

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END SEMESTER EXAMINATION NOV/DEC-2023

First Semester

M.Sc MATHEMATICS

CORE – II REAL ANALYSIS –I

Time: Three Hours **Maximum: 75 marks**

SECTION A – (15 x 1 = 15 marks)

ANSWER ALL QUESTIONS

1. If the function $f(x) = x^2$ is integrable on $[0, a]$ then $\int_0^a f(x)\,dx =$

A 0 B a

C $\frac{a^3}{3}$ D a^2

2. The function $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is discontinuous at

A 1 B 0

C -1 D 2

3. $U(p, f, \alpha)$=

A $\sum_{i=0}^{n} Mi\ \Delta\alpha i$ B $\sum_{i=1}^{n} Mi\ \Delta\alpha i$

C $\sum_{i=1}^{n} mi\ \Delta\alpha i$ D All of the above

4. If P* is a refinement of P, then L(P, f, α)- L(P^*, f, α)

A ≥ 0 B ≤ 0

C $< \frac{1}{n}$ D $> \frac{1}{n}$

5. Let f:[0,1]→ R be defined by $f(x) = \begin{cases} 1\ if\ x \in Q \\ 0, otherwise \end{cases}$ then

A f is Riemann integrable
B f is not Riemann integrable
C f is rectifiable
D none of these

6. If f ∈ $R(\alpha)$&g ∈ $R(\alpha)$ then fg is

A Riemann integrable
B Not Riemann integrable
C rectifiable
D none of these

7. If $\int_a^{b^-} f d\alpha = \int_{a^-}^{b} f\ d\alpha$ then f is called

A Riemann integrable
B partition
C Refinement
D none of these

8. P^* is the common refinement of two partitions P_1 and P_2

A $P^* = P_1\ U\ P_2$
B $P = P_1\ U\ P_2$
C $P = P_1^*\ U\ P_2$
D All of the above

9. If $S_{m,n} = \frac{m}{m+n}, \begin{matrix} m = 1\ to\ \infty \\ n = 1\ to\ \infty \end{matrix}$ find $\lim_{n\to\infty}[\lim_{m\to\infty} S_{m,n}] =$

A 0
B -1
C 2
D 1

10. If$f_n(x) = \frac{\sin nx}{\sqrt{n}}$(x real , n=1,2,.....).
&$f(x) = \lim_{n\to\infty} f_n(x)$ then $f'(x) = ?$

A 1
B 0
C ∞
D cannot be determined

11. $\int_a^b \frac{dx}{(x-a)^n}$ and $\int_a^b \frac{dx}{(x-b)^n}$ converges if

A n<1
B n>1
C n=1
D n = 2

12. If $f_n(x) = \frac{1}{nx+1}$ ($0 < x < 1$, $n = 1,2,3, \ldots$) then $f_n(x) \to 0$ monotonically in

A $[0\ \ 1]$
B $[0\ \ 1)$
C $(0\ \ 1)$
D R

13. If $f(x) = \begin{cases} x^{10} - 1, x \le 1 \\ x^2 - 1\ ,\ x > 1 \end{cases}$ then f is discontinuous at x=

A 1
B 2
C 3
D None of these

14. If c is an isolated point of S then f is ______ at c

A Connected
B Continuous
C Discontinuous
D None of these

15. Any convergent sequence is a ________.

A not bounded sequence
B bounded sequence
C limit not unique
D None of these

SECTION B – (2 x 5 = 10 marks)

ANSWER ANY TWO QUESTIONS

16. Define absolutely convergent.
17. State Euler's summation formula.
18. Define measure zero.
19. Define Cauchy product of series.
20. State Weierstrass M-test.

SECTION C – (5 x 10 = 50 marks)

ANSWER ALL QUESTIONS

21. A If f be a bounded variation on $[a, b]$ and V be defined on $[a, b]$,
$V(x) = V_f(a, x)$ for $a < x \le b, V(a) = 0$ then prove that
i) V is an increasing function on $[a, b]$
ii) $V - f$ is an increasing function on $[a, b]$.

OR

B State and prove Dirichlet's test.